Scattered Spherical Harmonic Approximation for Accelerated Volume Rendering

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1 Introduction

Recent computer graphics research has explored the use of spherical harmonics (SH) for global illumination style rendering. The lighting environment and the local visibility can be independently precomputed as SH, thus permitting near-instantaneous relighting of objects [Sloan et al. 2002].

A typical implementation stores a SH at each voxel of a volume and is expensive both in computation and in storage. We propose a method of sparsely populating a volume with SH and using least squares approximation to compute the SH at an arbitrary point.

2 Approximation Method

Density variations in the interior of volumes such as clouds or smoke tend to be smooth, thus relatively few samples need to be computed to capture the visibility detail. The variation is much greater near the edges of the volume, and therefore will require more explicit SH samples. It is also not necessary to sample in empty spaces far away from the volume, since these regions do not need to be lit. Such variable constraints suggested the use of a scattered data approximation technique.

Inspired by Arikan's use of scattered data interpolation of spherical harmonics [Arikan et al. 2005], we employ a least-squares fitting scheme. The objective is to find the coefficients of a degree 3 polynomial which approximates the SH coefficient's value [Press et al. 1992]. The minimization problem for each SH coefficient can be stated as:

$$min \sum_i \| f(\vec{x}) - y_m^l(\vec{x}_i) \|^2$$

This minimization problem can be solved with the following system:

$$\left(\sum_i p(\vec{x}_i) p(\vec{x}_i)^T \right) \mathbf{c} = \sum_i p(\vec{x}_i) y_m^l(\vec{x}_i)$$

where $\vec{x}_i$ is the location of each computed SH sample, $p(x)$ is the basis vector for the least-squares polynomial, and $\mathbf{c}$ is the vector of unknown coefficients. Note that matrix on left hand side contains only position information, thus using a solution method like LU decomposition permits reuse of factorization result when computing $\mathbf{c}$ for each SH coefficient.

3 Sampling

The underlying system used for modelling the volumes can suggest locations for computing SH samples. For example, if a collection of sphere primitives are used to fill a voxel grid, then computing a spherical harmonics sample at the center of each sphere can encode the visibility in the local region. For the general case we distribute points on a coarse isosurface around the density. A subset of these points are driven inside the surface, and some points are placed outside the surface to suppress oscillations from the fitting polynomial.

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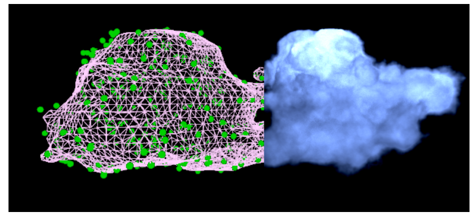

Figure 1: Sampling locations in green, and the underlying cloud isosurface.

4 Conclusions

Our results demonstrate that the qualitative difference in the rendered images is minimal. The approximation method successfully captures the subtle environmental lighting qualities present when rendering with computed spherical harmonics. In our method the memory use is not determined by the grid resolution, but the desired quality of output. The volume in Figure 2(a) required 30 MB of storage, and required 47 minutes of precomputation. The volume in Figure 2(b) using our approximation method requires 28 kB of storage, and 55 seconds of precomputation. The cache efficiency gained by repeatedly accessing a relatively small number of floating point numbers leads to a 4-fold rendering speedup, when compared to trilinear interpolation with a naive grid data structure.

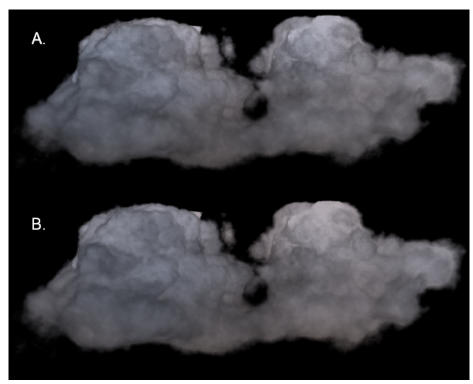

Figure 2: (A) A 204x173x321 cloud rendered with dense spherical harmonic sampling. (B) Same cloud rendered with approximated spherical harmonics.

References

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